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Searches for Optical Evidence of Galaxy Evolution

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Searches for optical evidence of galaxy evolution

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With improved photographic techniques and fast plate-measuring machines, we are now able to use deep photographic plates for statistical studies of faint galaxies. The evolutionary effects detectable with these samples are expected to be significant. Two such effects are currently being monitored by using P.D.S. and Cosmos scans of Schmidt and 4 m plates. First, number–magnitude counts are being used to constrain luminosity evolution, and secondly, correlation analyses of galaxy positions can be used to test physical models of galaxy clustering over large look-back times. The data can be reliably studied to magnitudes as faint as $J \approx 24.5$. Interpretation of the results is hampered by our poor knowledge of the intrinsic properties of normal galaxies.

1. Introduction

My review is concerned only with evolutionary studies that are possible with direct photographic plates. A complementary review on spectroscopic studies of individual objects at large red shifts has been given by Spinrad (1977) and I shall only make occasional references to that aspect. The ease with which properties of large numbers of faint images can now be determined is going to make the statistical approach increasingly important.

Much of the recent advance in photographic performance is due to the fine-grain emulsions now in use and the improved sensitization techniques developed at many observatories (Sim *et al.* 1976). Some idea of the progress can be seen by casual comparison of the new S.R.C. IIIa-J southern sky survey with its northern Palomar equivalent. In good observing conditions stars as faint as $J = 23$ can be reliably detected in 60 min with the U.K. Schmidt telescope (U.K.S.T.).

The combination of high quality panoramic plates and fast digital microdensitometers is a powerful research tool. The Schmidt samples, in particular, are ideal for statistical studies such as number–magnitude counts and correlation analyses of image positions. I shall deal with these topics and their relevance to galaxy evolution in detail. The conclusion is that the data are good enough to detect evolution, but that successful interpretation is severely limited by insufficient knowledge about galaxy properties at red shifts near zero.

2. Some typical data samples

The night sky brightness and the atmospheric 'seeing' make only some fraction of the total light of an extended image discernible above the plate noise. Consequently it becomes difficult to define a limiting integrated magnitude for galaxies on deep plates. In practice, images are detected by virtue of their central surface brightness (after seeing degradation) and their apparent size at some threshold brightness above the night sky. Typical threshold levels for detecting images and excluding noise are in the range 3–10 % above the sky (about 25.0–26.5 mag arcsec^{-2}). For U.K.S.T. plates scanned by the Cosmos machine in Edinburgh (Pratt 1977) the Durham group accepts only images with angular diameters $\theta > 4.3''$ at this threshold since star–galaxy separation becomes difficult below this limit. The greater plate scale of the Anglo-Australian telescope (A.A.T.) enabled Peterson *et al.* (1979) to study images reliably to $\theta \approx 1.4''$ on a 4 m plate taken during exceptionally good seeing conditions.

The question of whether a galaxy at a certain red shift is included in such machine-measured samples was considered in detail by Ellis *et al.* (1977). They developed a technique for predicting the distribution of recorded galaxies with depth. The code they used distributes galaxies homogeneously in co-moving space according to a Schechter-type luminosity function (Felten 1977) split into six morphological types, each with its own K-corrections (Pence 1976), evolutionary corrections and surface brightness profiles. The code allows for a Gaussian point-spread function (the seeing) and a minimum angular diameter above some isophote.

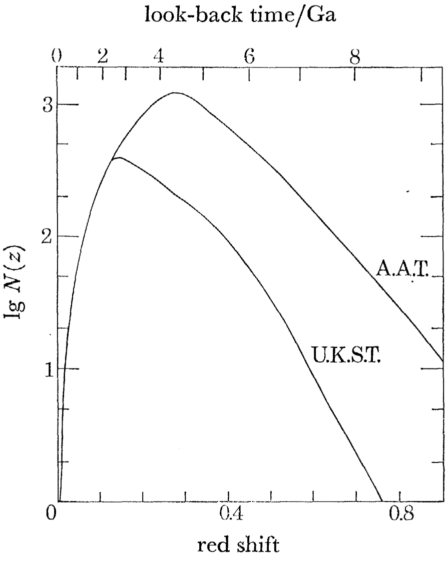

FIGURE. 1. Expected red shift distributions for galaxies detected with typical A.A.T. and U.K.S.T. data samples. The number refers to the differential count per square degree in a 0.01 interval in red shift. $\mu_J = 26.5$ mag arcsec^{-2}; seeing f.w.h.m. $= 0.8''$; $\theta_{\text{lim}} = 4.3''$ (U.K.S.T.) $= 1.4''$ (A.A.T.).

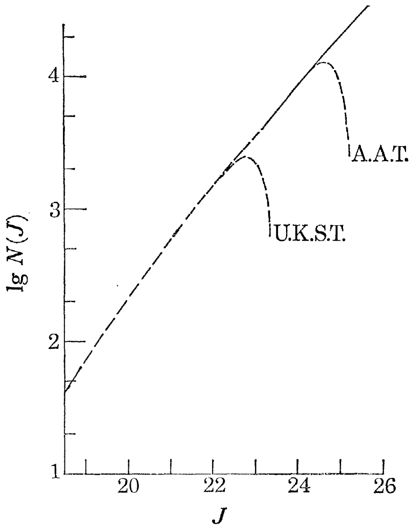

FIGURE 2. Expected differential number counts per square degree in 0.5 magnitude intervals as a function of magnitude J for galaxies on typical A.A.T. and U.K.S.T. data samples. The solid line represents the expectation for an idealized plate with no surface brightness losses. The broken curves deviate from this line at the completeness limit. All curves assume no luminosity evolution. Conditions as for figure 1.

In figure 1 I have used this code to predict the expected depths reached with high quality U.K.S.T. and A.A.T. J plates scanned respectively with Cosmos and a P.D.S. machine. Although shallower, the Schmidt plates cover a substantially larger area (the unvignetted field of the U.K.S.T. is *ca.* 20 deg^2). These plates need to be scanned at a resolution of better than $1''$ if they are to be fully exploited. The vast quantity of data that this generates restricts large-area scanning to fast microdensitometers with on-line processors such as Cosmos and A.P.M. (Kibblewhite *et al.* 1975); 4 m plates reveal substantial numbers of galaxies at red shifts in the range 0.2–0.7, sampling epochs up to 6 Ga ($= 6 \times 10^9$ years) ago. Over such timescales we expect significant luminosity evolution (see §3).

Figure 2 shows the expected J number–magnitude counts (assuming no luminosity evolution) for the same two data sets. They are compared with counts for an idealized plate without night sky or seeing losses. The point at which the real samples depart from the idealized curve defines the 'completeness limit'. The overall gain of the 4 m plates is *ca.* 2 magnitudes, the completeness limit for the best plates being $J = 24.5$. The reality of such faint images on these plates has been very clearly demonstrated by Peterson *et al.* (1979). They compared the A.P.M. output with a high contrast print of the plate and showed that images as faint as $J = 25$ were well above the plate noise.

Further details of current techniques for handling machine-measured data from photographic plates can be found in Godwin (1976), Kron (1978) and Shanks *et al.* (1979).

3. Evolutionary models

The evolution of the integrated light of a galaxy follows from an understanding of the evolution of a mixture of stars on the Hertzsprung–Russell diagram. The situation is complicated, however, by the possibility of a continuous birth of new stars and by enrichment of the gas by processed material. In attempting to simplify this complex subject for observers like myself I shall list some basic features and assumptions. The interested reader is advised to consult a more specialist review (e.g. Tinsley 1968, 1974) for finer details.

(i) The star formation rate (s.f.r.) is expected to depend at least linearly on the gas density (Larson 1977).

(ii) Given a mass of gas, some stellar mass distribution, known as the initial mass function (i.m.f.), eventually emerges. It is often assumed to be a power law:

$$\mathrm{d}N/\mathrm{d}M \propto M^{-(1+x)}.$$

As massive stars evolve more quickly, x determines the *rate* of change in the luminosity of a stellar population; x can be derived empirically in our Galaxy from composite mass functions of star clusters, or locally from the stellar luminosity function. Initial studies by Salpeter (1955) gave $x = 1.35$ for the solar neighbourhood, a value that has been widely used. Freeman (1977) has provided evidence for a *distribution* in x $(0 \lesssim x \lesssim 2)$.

(iii) The Hubble sequence of galaxy types can be fairly well explained simply in terms of different star formation rates (Baade 1963). In elliptical and S0 galaxies the s.f.r. was rapid, depleting the gas supply very quicly. The present population is red and has a high mass:light ratio. This type of galaxy was much brighter in the past by an amount depending on the epoch of this 'inital burst' (i.b.) of star formation. In spirals the blueness and presence of gas indicates star formation is still active. Consequently the s.f.r. must be more continuous; spirals will have looked much the same in the past as they do now.

There are four critical parameters in models of galaxy evolution: the rate of star formation, the i.m.f., the epoch of galaxy formation, z_f, and the cosmological model.

The red shift of galaxy formation, z_f, is believed to lie in the range 1–10. If galaxies formed before $z = 10$, their internal velocity dispersions would reflect the greater cosmic density and would be significantly higher than observed (Ostriker 1977). If $z_f < 1$ we should have probably stopped 'searching' for evolution many years ago, since for a small z_f, young galaxies are seen at brigher apparent magnitudes.

In analysing number–magnitude counts and magnitude–red shift (Hubble) diagrams, it has become well known that luminosity evolution plays a more important role than the cosmological model. One fact often overlooked, however is that evolutionary predictions can only be made once H_0 and q_0 are specified; stellar timescales (years) have to be linked to cosmological timescales (red shifts). In particular if q_0 is large, luminosity distances are decreased and the evolution becomes larger. Here I shall assume that $H_0 = 50$ and $q_0 = 0.05$.

Tinsley's earlier models (1977) used a rapidly decreasing s.f.r. for *all* types including spirals. Consequently as the proportion of spirals in the field is very high, it was predicted that young spirals (and ellipticals) would be seen in large numbers on deep plates. These models are now known to be unrealistic as they make present day spirals too red. Faint galaxy counts and extragalactic background light measurements (Dube *et al* 1977) also showed that the evolution could not be so strong.

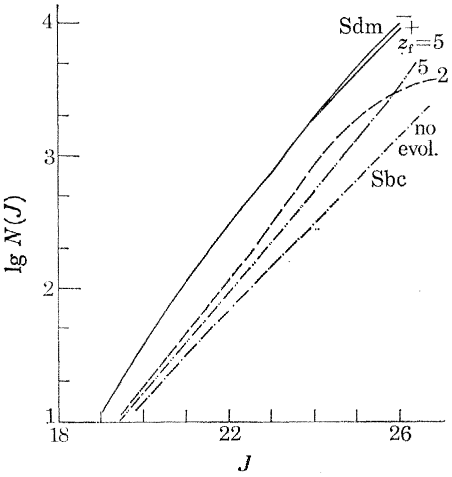

FIGURE 3. Expected differential number counts per square degree in 0.5 magnitude intervals as a function of magnitude J for spiral galaxies. The solid lines refer to predictions with (+) and without (−) luminosity evolution for types Sdm. The dashed series refer to types Sbc (for two formation red shifts, z_f).

The models that I shall use here assume a *steady* s.f.r. for spirals (Tinsley 1978). Figure 3 shows the predicted J number–magnitude counts for types Sbc and Sdm for no evolution and for continuous star formation with $z_f = 2$ and 5. Over the magnitude range available for counting the effect for the late types is very small. This is important because these spirals dominate the field counts.

Ellipticals are formed with an i.b. and, in addition to varying z_f, Tinsley has kindly supplied some $N(J)$ predictions where the duration of the star forming epoch is also varied.

Extended epochs of star formation might be expected if ellipticals formed gradually by a sequence of mergers (Tinsley & Larson 1979; Tinsley, this symposium). One might not expect to see the burst at all in optical wavelengths either because of dust absorption in primaeval galaxies or because much of the energy would be in the infrared. Figure 4 shows that the bursts produce humps in the $N(m)$ plot; the width of the hump is decreased for the short bursts, and the position (i.e. magnitude) depends on z_f as mentioned earlier. If it were possible to separate ellipticals from spirals at these magnitudes we should certainly be able to constrain these models. This would be an extremely difficult task, however, for the distant ellipticals at $z > 2$ might be indistinguishable from stars.

To be seen as a galaxy in the present 4 m samples, a young elliptical would have to be at least 30 kpc across at the $\mu_J = 26.5$ mag arcsec^{-2} isophote (approximately independent of z_f).

This may or may not be possible depending on the nature of the primaeval object (see Sunyaev *et al.* 1978).

It therefore appears that a large fraction of the galaxies are not expected to evolve very much on number–magnitude plots. Because of this and because of the homogeneity of giant ellipticals as standard candles, searches for evolution have been primarily conducted by using magnitude and colour–red shift diagrams for brightest cluster members. The selection effects operating here are, in my opinion, insuperable and the availability of magnitudes (as opposed to faint spectra) is forcing us to contend with number–magnitude rather than magnitude–red shift plots. The situation is not quite so depressing, for if young ellipticals *are* recognizable as galaxies, their evolution is, in most cases, strong enough to be seen in the total counts. Furthermore, colour data for faint galaxies are quickly becoming a powerful weapon for evolutionary studies (Kron 1978; see also §4). With the space telescope there is a real possibility that the angular resolution will be sufficient to classify galaxies (albeit crudely) at the magnitudes of interest (Fong 1979).

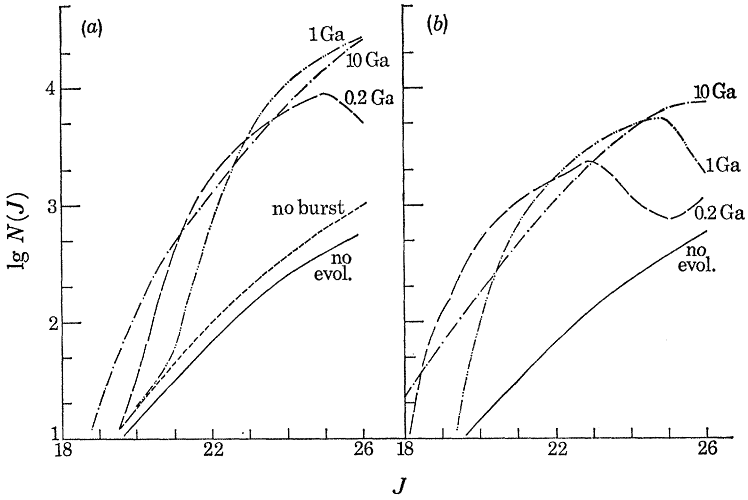

FIGURE 4. Expected differential number magnitude counts per square degree in 0.5 magnitude intervals as a function of magnitude *J* for elliptical and S0 galaxies. The curves refer to predictions based on early rapid star formation models at formation red shifts (*a*) $z_f = 5$ (*b*) $z_f = 2$. The curves differ in the duration of this era as marked. The 'no burst' curve refers to the case where the burst of star formation is obscured, e.g. by dust.

4. NUMBER–MAGNITUDE COUNTS OF FAINT GALAXIES

Number–magnitude counts were first used by Hubble (1926) to demonstrate the large-scale homogeneity of the Universe. For faint magnitudes ($m > 18$) deviations from a pure Euclidean relation can be used to study luminosity evolution, K-corrections, the galaxy luminosity function and cosmology (in decreasing order of sensitivity). Number counts are now used to search for luminosity evolution under the (incorrect) assumption that the other parameters are well known.

Reliable magnitude counts are very difficult to obtain. For very faint galaxies, automated methods are essential because there are problems of definition and completeness (see §2). The theoretical models must either be used to predict exactly what is measured (e.g. isophotal magnitudes), or one must use only those brighter images unaffected by selection effects of size and surface brightness.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Counts have been reported by various observers using different techniques. Machine-measured deep counts have been reported only by Kron (1978) and Peterson *et al.* (1979). Figure 5 shows a comparison of all counts fainter than $J = 19$ (apart from those of Karachentsev & Kopylov (1977) who collated counts by eye from different sources – an unreliable procedure). One important feature of the Peterson counts (labelled A.A.T. and U.K.S.T.) is that there is reasonable continuity between the two sets. This is encouraging for the data reduction techniques were quite different. The U.K.S.T. data were scanned by Cosmos, whereas the A.A.T. data were first measured by a P.D.S. machine and subsequently analysed with Kibblewhite's A.P.M. software.

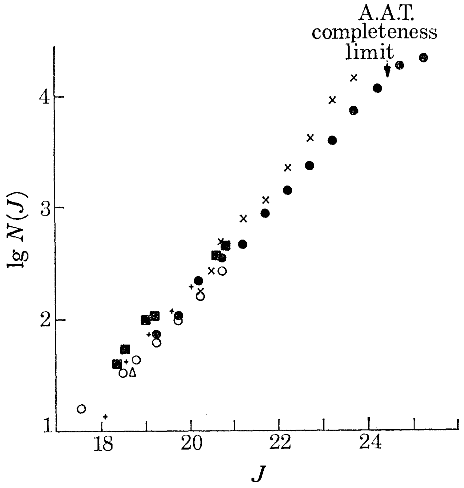

FIGURE 5. Differential number counts of galaxies per square degree in intervals of 0.5 magnitude as a function of magnitude J for various data samples: ●, A.A.T.; ○, U.K.S.T.; ×, Kron; +, Rainey; ■, Brown; △, Lick survey.

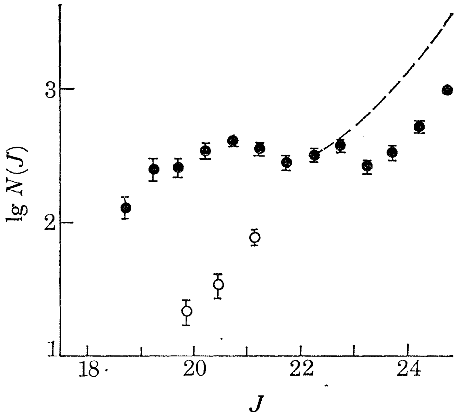

FIGURE 6. Differential number counts of objects not classified as galaxies per square degree in intervals of 0.5 magnitude as a function of magnitude J. The broken line corresponds to the upper limit to this population. The true stars end at $J \approx 22.5$ leaving an increasing population of faint compact images possibly quasars (○, published quasar counts).

Star–galaxy separation of Schmidt data is a painstaking procedure (Shanks 1979). Fortunately, for deeper counts Nature is on our side. The ratio of the number of stars to galaxies rapidly decreases when $J > 22$, an important fact independently discovered by Kron, by Karachentsev & Kopylov and by Peterson *et al.* This makes 4 m galaxy counts almost independent of the star–galaxy separation algorithm. Over the region where the separation is important, the best technique appears to be that which uses the surface brightness profile. Faint galaxies have flatter profiles than stars of the same magnitude as they are extended images. Successful classification at $J > 23$ requires exceptionally good seeing.

Figure 6 shows number–magnitude counts for the star-like images on Peterson's A.A.T. plate. They are important because they reveal the existence of a separate population of compact objects (fainter than the *bona fide* stars). It is unlikely that these are the distant ellipticals referred to in § 3 because the numbers involved are too small. It is more probable that they are a mixture of faint quasars and nearby dwarf ellipticals. The faint galaxy counts are not, incidentally, dominated by nearby dwarf galaxies, a fact demonstrated by Kron (1978). Some (highly uncertain) quasar counts mentioned in Smith (1978) are plotted for comparison purposes.

The galaxy counts in figure 5 are all in reasonable agreement (there is some uncertainty in the absolute magnitude scale which allows a horizontal shift of about 0.3 mag) except that

Kron's slope at $J = 23$ is 0.54 ± 0.01, steeper than that of Peterson *et al.* (0.455 ± 0.005). The corresponding slope for no evolution is only 0.40 (figure 2). The reason for the discrepancy between the observations is not clear; Kron attempts to measure total magnitudes – an easier quantity to model, though the errors then increase strongly with magnitude. The different magnitude scales and the Malmquist bias in Kron's data appear to explain some part of this discrepancy.

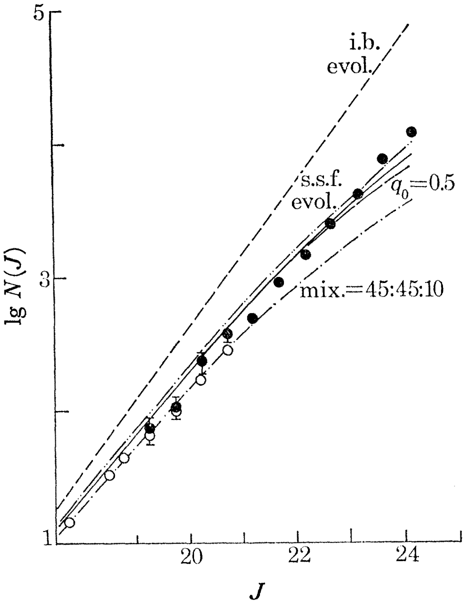

FIGURE 7. Differential number counts of galaxies per square degree in intervals of 0.5 magnitude as a function of magnitude J for data derived from A.A.T. (●) and U.K.S.T. (○) plates. The solid curve represents the expected number counts on the standard model which takes into account the appropriate threshold isophote, the minimum number of pixels per image, seeing, K-terms, luminosity function and a mixture of galaxy types (E+S0:Sab+Sbc:Scd+Sdm = 25 %: 25 %: 50 %). It assumes $q_0 = 0.05$ and has no luminosity evolution. The other dashed curves differ from the standard model in the variation of only one parameter as shown; i.b. has an initial burst of star formation for *all* types at a formation red shift of 6.2; s.s.f. has steady star formation for spirals with an initial burst (which remains invisible) at a formation red shift of 5 for elliptical and S0 galaxies.

Before claiming that evolution has been detected we should examine the uncertainty in the slope of the no evolution curve. In figure 7 the A.A.T. and U.K.S.T. counts are compared with predictions for various input parameters. The steeper slope could be consistent with no evolution if the K-corrections for spirals were overestimated over the red shift range 0.3–0.8 (rest wavelengths above 250 mm). Alternatively the evolution could be quite strong if the mix of types were more heavily weighted towards ellipticals. The exact interpretation of the data rests more on the intrinsic properties of galaxies than anything else. This problem will not disappear when the space telescope is launched; we should plan *now* to do a lot of basic astronomy on galaxy properties.

If the input physics is correct and we used the models supplied by Tinsley, the elliptical burst is fortunately expected to be seen above the overall field total, but the continuity in slope over a wide magnitude range in the data becomes hard to fit (see figure 8). The best fit to Kron's data has $z_f = 5$ with a burst of 1 Ga. Alternatively, if the galaxy mix were changed this model could be made to fit Peterson's data.

In addition to having rather more faint J galaxies, Kron also finds that the J–F colour becomes progressively bluer for $J > 22$. Spinrad claims that this is additional evidence for the detection of young ellipticals at $z_f = 2$, assuming that such images are recognizable as galaxies.

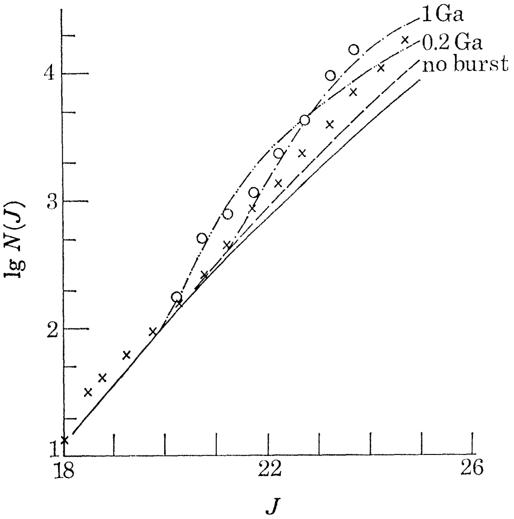

FIGURE 8. Differential number counts of galaxies per square degree in intervals of 0.5 magnitude as a function of magnitude J for the data of Kron (○) and Peterson *et al.* (×). The solid curve represents the expectation for the standard model without evolution. The dashed curves represent predictions on the steady star formation scheme with a formation red shift of 5. The curves differ in the duration of the star forming era for elliptical and S0 galaxies. The curves were derived from those in figures 3 and 4 together with other curves for intermediate galaxy types.

5. CORRELATION ANALYSIS OF DEEP GALAXY SAMPLES

An important application of deep data samples is the dependence of angular clustering with limiting magnitude. The dependence observed tests the applicability of a single physical model for galaxy clustering over very large volumes. With sufficient precision and a better knowledge of galaxy properties than is currently available, it might also be possible to place constraints on the dynamical evolution of large numbers of galaxies. On the gravitational instability theory of galaxy formation, such evolution is expected to be very small over our available look-back times. The importance of these studies is also in the application of our uncertain knowledge of galaxy properties, the results being used to constrain these uncertainties.

(*a*) *Small-scale clustering*

Clustering is usually discussed in terms of the two-point galaxy correlation function. In three dimensions the function $\xi(r)$ measures the excess probability of finding pairs of galaxies r Mpc apart. In two-dimensional projection the analogous (observable) function is $w(\theta)$. For a full description and set of definitions see Peebles (1973).

The relation between ξ and w is due to both geometrical and projection effects and was first discussed by Limber (1953). A relativistic version of his equations was derived by Phillipps *et al.* (1978). Provided the probability of detecting a galaxy for the sample in question is known (e.g. from figure 1), projection can be statistically allowed for and angular correlations can be compared at different depths (a technique referred to as 'scaling').

Schmidt plates are ideal for measuring correlation functions because large areas without Galactic obscuration gradients are needed to ensure a 'fair sample'. Correlation evolution (e.g. cluster collapse) is likely to be detectable, if at all, only on 4 m and space telescope pictures where large numbers of frames will need to be gathered to ensure a fair sample.

Figure 9 shows angular correlation functions for a wide range of samples, both shallow and deep. Correlations are less on deep surveys because fixed angles span larger distances where clustering is weaker, and because increased depth leads to more projected (line-of-sight) pairs. Over the range 0.05–9 h^{-1} Mpc the angular function is well fitted by a power-law of slope

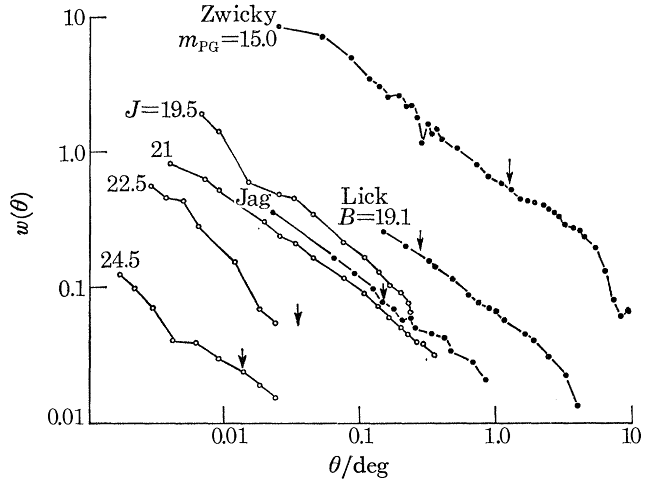

FIGURE 9. Correlation functions plotted against angular separation, θ, for some shallow and deep galaxy samples. Filled circles refer to values derived by Peebles, open circles to U.K.S.T. and A.A.T. plates analysed at Durham. The arrow marks the angle approximately equivalent to a separation of 1 h^{-1} Mpc.

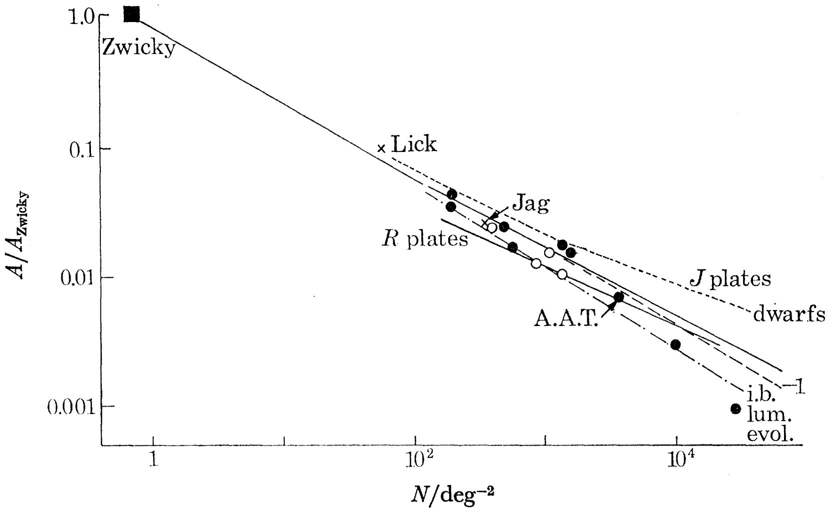

FIGURE 10. Amplitudes of −0.8 power-law fits to angular correlation functions (with respect to the amplitude for the Zwicky catalogue) plotted against surface density of galaxies for various data samples. Filled circles refer to U.K.S.T. and A.A.T. J plates, open circles to U.K.S.T. R plates, and crosses to data analysed by Peebles. The solid lines represent expectations for virialized clusters as seen in J and R bands; the curve labelled −1 allows for cluster collapse at a rate proportional to $(1+z)$; i.b. shows the effect of Tinsley's initial burst evolutionary models and the curve labelled 'dwarfs' shows the effect of including a large number of faint galaxies in the luminosity function.

ca. -0.8. The exact slope is uncertain; some of the deeper samples give flatter slopes which may be due to image confusion at very small scales.

Assuming that $w(\theta) \propto \theta^{-0.8}$, the correlation amplitudes at various limiting magnitudes can be compared with predictions that allow for projection. The projection factors are sensitive to the depth distributions and hence to the galaxy luminosity function and its evolution. The 'scaling' results are collated in figure 10.

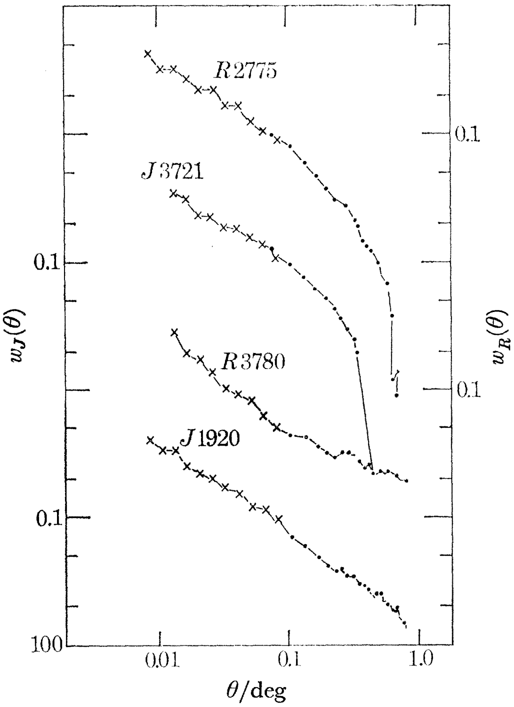

FIGURE 11. The angular correlation functions for four U.K.S.T. plates, two fields each photographed in J and R. The functions are separated in the ordinate for clarity. Plates of the south Galactic cap show a feature in the correlation function, whereas those of S.R.C. field 412 do not.

The solid lines ($\beta = 0$) represent models where clusters are in virial equilibrium; a different curve applies for red plates because of luminosity function differences. The degree to which cluster collapse would be detectable is represented by the line $\beta = -1$; here clusters collapse at a rate proportional to $(1+z)^{-\beta}$. The correlation amplitudes are derived from regions of the data where clusters are expected to be near-virialized and the U.K.S.T. data follow these $\beta = 0$ curves to within *ca.* 40% to $J = 21.5$. The deeper 4 m amplitudes get progressively more discrepant at fainter limits, an effect discovered in *small* Schmidt samples (Phillipps *et al.* 1978). Rather than invoke exotic mechanisms for this behaviour, I believe that it is an effect related to the tiny sample involved; the points are plotted as an appetizer for future 4 m results!

Two interesting by-products of figure 10 are the sensitivity of the scaling line to luminosity evolution (line labelled 'i.b.') and an extension of the luminosity function to $M_J = -14$ ($H_0 = 50$) (line labelled 'dwarfs'). Both weight the depth distribution, though in opposite ways. The absence of large numbers of dwarf galaxies per unit volume has been confirmed recently by Jones & Jones (1978).

(*b*) *Large-scale clustering*

On the gravitational instability picture no features are expected in the observed $w(\theta)$ unless previrialization has occurred in a closed Universe (Davis *et al.* 1977). As Efstathiou (1979) remarks, the *existence* of a feature would be difficult to reconcile with an open Universe. Such a feature is already claimed by Groth & Peebles (1977) to exist in the angular function for three local catalogues. Moreover, the feature, which manifests itself as a change in slope from -0.8 to *ca.* -2 at separations corresponding to *ca.* 9 h^{-1} Mpc, scales reasonably well between the catalogues. One disadvantage of local surveys, however, is that it is not clear how much of the apparent galaxy distribution is intrinsic and how much is affected by Galactic absorption and variations from plate to plate. At large scales, w is very small and severely affected by such technical problems. In the U.K.S.T. samples limited at $J = 21.5$ the feature should be *ca.* 0.6° where such problems might be expected to be minimal.

Figure 11 shows $w(\theta)$ for four U.K.S.T. samples (not scaled), two fields each in two colours, near the south Galactic pole. There is some evidence for a feature in one of these fields (both J and R) but *not* in the other. There is, however, a ridge of Galactic obscuration which can be clearly seen in the galaxy distribution for this field. The cosmologically important question of whether there is a feature at large scales must await further analysis of other U.K.S.T. plates now at Durham. The feature could also be checked reliably by using large complete red shift samples.

6. CONCLUSIONS

Have we detected evolution in galaxy luminosities? Probably, but its exact interpretation is still difficult and much work needs to be done before the age of galaxies can be measured. The present analyses must be repeated with other fields. The Durham group plan to embark upon colour–magnitude studies with A.P.M. by using good seeing A.A.T. plates; this will be an important check of Kron's exciting results.

More vital, however, is the need for a better understanding of the properties of normal galaxies. Hopefully we shall soon have reliable K-corrections from the I.U.E. satellite. The galaxy luminosity function, its normalization and division by morphological type, are all too uncertain. A large magnitude-limited sample complete with red shifts and types at magnitudes 16–17 would help enormously. In the years up to the launch of the space telescope it is essential that these basic details are supplied otherwise we shall be at a loss as to how to interpret counts and correlation functions to the 28th magnitude!

Data reduction of Schmidt plates at Durham owes much to the hard work of T. Shanks and H. MacGillivray. I also acknowledge useful discussions with D. Carter, G. Efstathiou, S. M. Fall, R. Fong, E. J. Kibblewhite, R. G. Kron and B. A. Peterson. Beatrice Tinsley is thanked for a generous supply of models specific to our projects.

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